National Research Council Canada | Conseil national de recherches Canada

Canada

Reprinted from

Canadian Journal of Forest Research

Réimpression du

Journal canadien de recherche forestière

# The power function as a simple stem profile examination tool

R. R. FORSLUND

Volume 21 • Number 2 • 1991

Pages 193–198

Printed in Canada / Imprimé au Canada

91-06810/



# The power function as a simple stem profile examination tool

R. R. FORSLUND

*Centre for Northern Forest Ecosystem Research, Ontario Forest Research Institute, Ontario Ministry of Natural Resources, c/o Lakehead University, Thunder Bay, Ont., Canada P7B 5E1*

Received May 2, 1990

Accepted August 23, 1990

FORSLUND, R. R. 1991. The power function as a simple stem profile examination tool. Can. J. For. Res. **21**: 193–198.

Formulae are presented for calculating the volume, surface area, centre of gravity, centre of mass, form, taper, and instantaneous slope of the power function. A profile plotting technique is also used that allows visual comparisons between average stem profiles independent of the size and taper of individual stems. The technique is used to demonstrate that the position near 30% of the height from the base of the stem is a position of form stability for volume estimation. Using the paracone profile model (a power function midway between a paraboloid and a cone) with the diameter at the 30% position and the total stem height, individual stem volume estimates within 10% of the true volume (95% confidence) are obtained once again using a sample of 50 yearling aspen (*Populus tremuloides* Michx.). The plotting technique is then applied to the aspen sample, and the average profiles support the hypothesis that juvenile stems may begin life as paraboloids. The average centre of gravity, the average centre of mass, the sectional form, the average form, and the taper of the stems are also examined and are shown to be quantitative indicators of the plotted profile characteristics.

FORSLUND, R. R. 1991. The power function as a simple stem profile examination tool. Can. J. For. Res. **21** : 193–198.

Des formules sont présentées pour calculer le volume, la surface terrière, le centre de gravité, le centre de masse, la forme, le défilement et la pente instantanée de la fonction de puissance. Une technique de tracé de profils est aussi utilisée qui permet des comparaisons visuelles entre les profils moyens des tiges qui sont indépendants de la dimension et du défilement individuel des tiges. La technique est utilisée pour démontrer que la position située près de 30% de la hauteur à partir de la base de la tige, est une position stable pour l'estimation du volume. Utilisant le modèle de profil paracone (une fonction de puissance à mi-chemin entre la paraboloïde et le cône) avec le diamètre à la position 30% et la hauteur totale de la tige, des estimations individuelles de volume à l'intérieur de 10% du vrai volume (intervalle de confiance à 95%) sont obtenues à partir d'un échantillon de 50 jeunes tremble (*Populus tremuloides* Michx.). La technique du graphique est alors appliquée à l'échantillon de tremble et les profils moyens supportent l'hypothèse que les tiges juvéniles évoluent selon des paraboloïdes. Le centre le gravité moyen, le centre de masse moyen, la forme sectorielle, la forme moyenne et le défilement des tiges sont aussi examinés et se montrent des indicateurs quantitatifs des caractéristiques graphiques des profils.

[Traduit par la Rédaction]

## Introduction

Many mathematical functions are being used to define the average profile of a group of stems. Individual stem profiles, however, exhibit deviations from a smooth curve. Some of the variation is due to form, whereas some is due to measurement errors. Ideally, if a stem irregularity, caused by swelling, wounds, or other defects, occurs at an upper measurement position on the stem, then diameters should be measured equidistant above and below the irregularity. In practical sampling for stem analysis, however, a disk is cut at the required position, and diameters are measured directly from this disk. The resultant diameters are often not representative of the average profile at that position. Add measurement errors to the situation, and we must ask if we are measuring a sponge with vernier calipers. Regression models using complex sigmoidal functions will provide better "fits" for individual profiles than nonsigmoidal functions such as the power function. However, does the resultant model accurately describe the average profile of a group of stems, or would a simpler function be adequate for many purposes?

Although sigmoidal functions fit the individual profile better, they have a disadvantage. Simple formulae for the volume of the rotated function, for example, are difficult to derive and usually require iterative techniques to solve complex integrals. For example, Forslund (1982) attempted to use the following sigmoidal function to define the profile:

$$[1] \quad Y = (1 - X^A)^{(1/B)}$$

where $A$ and $B$ are the exponents of $X$ and $Y$, respectively, $X = D_Y/D_0$, $D_0$ is the basal diameter, $D_Y$ is the diameter at a relative height, $Y = h/H$, $h$ is the height to the measurement position from the base of the stem, and $H$ is the total stem height.

When eq. 1 is integrated, the resultant formulae involve gamma functions that are difficult to deal with even in the simplest of models. As a result, this model was reduced to the following inverted power function by letting $B = 1$:

$$[2] \quad Y = 1 - X^A$$

A more general form of eq. 2 relating any two diameters, $D_{Y1}$ and $D_{Y2}$, at relative heights of $Y1$ and $Y2$, respectively (Ormerod 1973), is

$$[3] \quad \frac{D_{Y1}}{D_{Y2}} = \left[\frac{1 - Y1}{1 - Y2}\right]^{(1/A)}$$

Before continuing the discussion, it is necessary to define the terms form and taper. Form is defined as the geometrical shape of the stem profile and is quantified by the exponent $A$ in either eq. 2 or 3. The form is neiloidal, conical, paraconical, and paraboloidal when $A = 0.667$, 1.0, 1.5, and 2.0, respectively. The term paracone was previously used (Forslund 1982) to describe the form that lies between a paraboloid and a cone and has a value of $A$ of 1.5. Taper is defined as the change in height between two measurement positions divided by the corresponding change in radius.

Printed in Canada / Imprimé au Canada

This definition is identical with the mathematical definition of slope. Similar definitions of form and taper were used by Gray (1956), Newnham (1988), and others.

Historically, the power function has found widespread use in many fields of science. In classical mechanics, for example, a projectile falling under the influence of gravity follows a parabolic trajectory, which is a power function using an exponent of $A = 2$. Over the last decade, the power function has found extensive use in the fractal geometry of Mandelbrot (1982) and is also the traditional function for allometric analyses in biology (Gould 1966). The attraction of the power function lies in its simplicity and versatility, and it continues to find new applications. It has appeared in forestry literature in articles dealing with the $-3/2$ power law for self-thinning (Zeide 1987 for example). It has also been used to approximate the stem profile by authors such as Winters (1932), Ormerod (1973), Ueno (1978), Forslund (1982), Reed and Byrne (1985), Real et al. (1989), and others. The use of the power function to describe the form of the stem profile in the lower, mid, and upper sections, as neiloidal, paraboloidal, and conical, respectively, is well known. Recently, Newnham (1988) and Kozak (1988) have extended the sectional usage by allowing the exponent of the power function to be a function itself, thus creating a continuum of forms up the stem.

One aspect of the stem that has not been addressed is the basic form of a stem profile as it begins life. The thought was that perhaps trees start life as paraboloids and then, as various factors influence them, this basic form is affected. If this hypothesis is true, then a consistent juvenile form will provide a starting place to determine which factors affect this basic form. The form variation can then be studied, as the stems age under a variety of conditions and constraints, to develop a model of the stem profile over time.

The intent of this paper is to present and discuss several mathematical relationships associated with the power function and to illustrate the application of these formulae by examining the paraboloidal juvenile stem form hypothesis.

## Mathematical theory and formulae

*Preliminary formulae*

Since the basal diameter of a stem is highly variable due to basal flare, all of the following formulae were related to upper stem diameters using eq. 2 rearranged as follows:

$$[4] \quad D_Y = (1 - Y)^{(1/A)} D_0$$

Solving eq. 4 for $D_0$, results in

$$[5] \quad D_0 = (1 - Y)^{(-1/A)} D_Y$$

The following formulae are derived using eq. 2, and its counterparts, eqs. 4 and 5. All measurements are assumed to be in the same units for this section of the article, although incorporating unit conversion factors does not affect the relationships. All calculations, such as volume, are assumed to be either outside bark, if outside bark diameters are taken, or inside bark, if inside bark diameters are taken.

*Volume and surface area*

By integrating eq. 2, the total stem volume ($V$) given the diameter $D_Y$ at a relative height $Y$ (Forslund 1982) is

$$[6] \quad V = \frac{A}{(A + 2)}(1 - Y)^{(-2/A)} \frac{\pi D_Y^2 H}{4}$$

From eq. 6, the volume of the paracone ($A = 1.5$) (Forslund 1982) is

$$[7] \quad V = \frac{3}{7}(1 - Y)^{(-4/3)} \frac{\pi D_Y^2 H}{4}$$

The merchantable volume ($V_m$) to an upper diameter ($D_U$), which is usually a constant value, at a relative height ($U$), given the total stem volume ($V$) from eq. 6 for a given form ($A$) (Forslund 1982) is

$$[8] \quad V_m = V - \frac{A}{(A + 2)} \frac{\pi D_U^2}{4} H(1 - U)$$

The surface area (SA) of the power function given the diameter $D_Y$ at a relative height $Y$, can be derived as

$$[9] \quad SA = \frac{A}{(A + 1)}(1 - Y)^{(-1/A)} \pi D_Y H$$

*Centre of gravity and centre of mass*

The centre of gravity (COG) for a given form $A$ (Forslund 1982) is

$$[10] \quad COG = \frac{A}{2(A + 1)}$$

The centre of mass (COM) for a given form $A$ (Wood et al. 1990) is

$$[11] \quad COM = 1 - 2^{-(A/(A + 2))}$$

It is important to note than when any nonsymmetrical solid of uniform density is positioned horizontally and is under the influence of gravity, the COG and COM are distinctly different. Although the density of the wood varies along the length of the stem, uniform density is assumed in eqs. 10 and 11 for the purposes of determining average stem form. The calculated COG and COM for the cone, paracone, and paraboloid using eqs. 10 and 11 are given in Fig. 1. The COG and COM can also be calculated using calculus and iterative techniques with the diameter–height measurements up the stem.

*Quantitative measures of stem form*

The sectional form ($A_{SEC}$) of the stem can be derived from eq. 3 (Newnham 1988 and others):

$$[12] \quad A_{SEC} = \frac{\ln((1 - Y1)/(1 - Y2))}{\ln(D_{Y1}/D_{Y2})}$$

This form value is reminiscent of the fractal dimension used in fractal geometry (Mandelbrot 1982), in which he expresses a ratio of logarithms similar to eq. 12. Diameters at fixed heights such as breast height should be avoided when using eq. 12 to estimate sectional form since the resultant sections are then dependent on the height of each stem. This is especially true for smaller stems, where the relative position of breast height varies significantly.

The form ($A_{COG}$) given the COG was derived by rearranging the terms of eq. 10 into

$$[13] \quad A_{COG} = \frac{2COG}{(1 - 2COG)}$$

The form ($A_{COM}$) given the COM was derived by rearranging the terms of eq. 11 into

$$[14] \quad A_{COM} = \frac{-2 \ln(1 - COM)}{\ln(2) + \ln(1 - COM)}$$

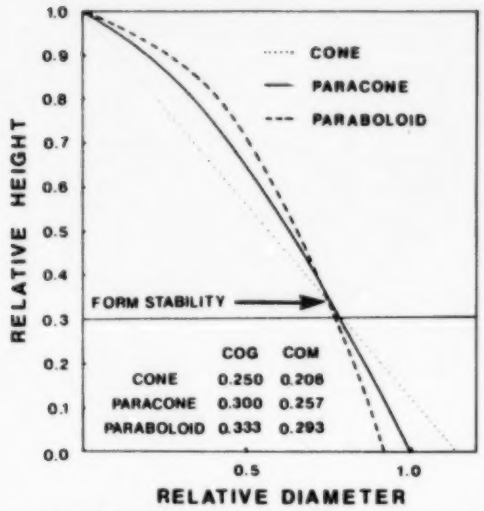


FIG. 1. Relative profile overlays of the cone, paracone, and paraboloid of equal volume using the profile plotting technique. COG, centre of gravity; COM, centre of mass.

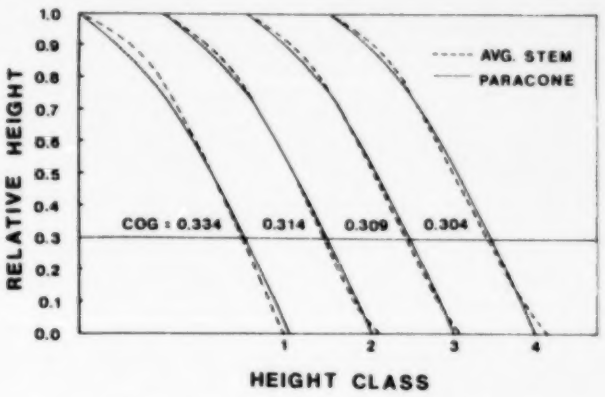


FIG. 2. The profile plotting technique, comparing the average profiles and centre of gravity (COG) of 50 yearling aspen relative to the paracone. The aspen are stratified into four height classes from short to tall stems.

Since the COG and COM are for the entire stem, the form values in eqs. 13 and 14 reflect an overall or average stem form rather than the form of a section of the stem as was given in eq. 12.

*The taper and slope*

A measure of stem taper was needed without having to take additional measurements on the stem. Since $D_{0.3}$ and $H$ are measured to provide estimates of volume using eq. 7, a taper formula was derived incorporating these two measurements. Once again, a relative height position is used, rather than a fixed location such as breast height, so that taper can be compared between individual or groups of stems, independent of the size of each stem. Using the height from the 30% position to the tip (0.7 $H$), divided by the radius ($D_{0.3}/2$) at this position, the taper (T) is

$$[15] \quad T = \frac{1.40H}{D_{0.3}}$$

The instantaneous slope ($S_Y$) at a relative position ($Y$) is found by taking the derivative of eq. 2:

$$[16] \quad S_Y = -A(1 - Y)^{((A-1)/A)}$$

Similarly, the instantaneous slope ($S_h$) at a fixed height ($h$) on the stem is

$$[17] \quad S_h = -A\frac{2(H - h)}{D_Y}$$

Notice that the term $2(H - h)/D_Y$ is the actual taper between position $h$ and the top of the stem. If $h = 0.3H$, then eq. 17 is related to the taper presented in eq. 15 by the relationship

$$[18] \quad S_{0.3H} = -AT$$

It is significant to note that the slope is the product of the form ($A$) and the taper ($T$).

## Applications of the power function

*Data*

To illustrate the use of the previous power function formulae, and to test the hypothesis of a paraboloidal juvenile stem form, approximately 100 one-year-old aspen (*Populus tremuloides* Michx.) in the vicinity of Thunder Bay, Ontario, were cut at the root collar. Stems with defects or disease were rejected, thus reducing the sample to 50 quality stems. The buds and branches were removed, and each stem was balanced horizontally to obtain its COG. Diameters at 10% intervals of the total height starting at the base of the stem were measured with vernier calipers, and the total stem height was taken. The heights ranged from 42 to 178 cm. If a swelling occurred at a measurement location, then two diameters were taken equidistant from the position and an arithmetic average was calculated.

*Stem profile plotting technique*

Forslund (1982) presented a technique that converts individual stem profiles to relative profiles. The technique allows visual examination of average form variation between groups of profiles independent of stem size and taper. The paracone profile is used as a reference curve to observe variation in the average stem profile. The height to each measurement position is changed to a relative value by dividing by the total stem height. Relative diameters are determined using a slightly different method of calculating an expected basal diameter for each stem using the volume formula for the paracone (eq. 7) and dividing this value into the actual diameters up the stem. Using an expected rather than an actual diameter at a selected relative position on the stem prevents the situation where the profile is forced through a value of 1 at the selected position. A compounding problem occurs when the selected position is at a fixed height. For example, several authors such as Newnham (1988) and Czaplewski (1989) use a form quotient in which all upper diameters are made relative to dbh. The use of a reference diameter at a fixed rather than relative position on the stem forces the relative height of each reference diameter to vary between stems. This results in inconsistent relative profiles, which causes the plotted average relative profile to be incorrect. This is especially true for smaller stems where the relative position of dbh varies greatly.

To examine the paraboloidal juvenile stem hypothesis and illustrate the practical use of the profile plotting technique, the 50 aspen stems were arranged in ascending order by height and grouped into classes 1 to 4, each with a sample

TABLE 1. Freese's percent $\chi^2$-values for volume prediction of the aspen sample using selected forms at each relative location on the stem

| | % $\chi^2$-values for relative heights: | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *A* | 0.0 | 0.1 | 0.2 | 0.3 | 0.333 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
| 2/3 | 66 | 55 | 32 | 7 | 20 | 57 | 141 | 288 | 600 | 1546 | 7560 |
| 1.0 | 33 | 33 | 23 | 11 | 6 | 13 | 39 | 78 | 142 | 293 | 868 |
| 1.5 | 21 | 12 | 10 | 9 | 8 | 8 | 11 | 20 | 27 | 49 | 131 |
| 2.0 | 45 | 21 | 11 | 7 | 6 | 10 | 18 | 30 | 38 | 44 | 46 |
| 2.5 | 67 | 35 | 21 | 9 | 6 | 9 | 22 | 38 | 53 | 65 | 71 |
| 3.0 | 85 | 48 | 29 | 12 | 8 | 9 | 24 | 44 | 61 | 77 | 89 |

NOTE: The $\chi^2$-values imply that volume estimates will be within this percent of the true volume with 95% confidence. The exponent (*A*) of the power function determines the form of the volume model (eq. 6).

TABLE 2. Average sectional form ($A_{SEC}$) over the height classes for the aspen sample

| Height class | $A_{SEC}$ for each 10% section[a] | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1 | 1.29 | 2.49 | 1.80 | 2.03 | 2.00 | 2.27 | 1.94 | 2.23 | 2.67 |
| 2 | 1.01 | 1.58 | 1.93 | 1.60 | 1.83 | 1.72 | 1.53 | 2.53 | 2.43 |
| 3 | 1.32 | 1.37 | 1.68 | 1.60 | 1.70 | 1.40 | 1.76 | 1.90 | 2.49 |
| 4 | 0.87 | 1.33 | 1.32 | 1.75 | 1.48 | 1.58 | 1.74 | 1.95 | 2.36 |
| All | 1.12 | 1.70 | 1.68 | 1.75 | 1.75 | 1.75 | 1.75 | 2.15 | 2.49 |

[a]Section 1 is from 0 to 10% of the height from the base, section 2 is from 10 to 20%, etc.

size of 12 or 13 stems. Figure 2 shows the resultant average profiles and illustrates generally paraboloidal stem forms. However, a progressive form variation over the four height classes is evident from the slight increase in basal flare. This visual support of the paraboloidal juvenile form will be quantified later when form and taper are discussed.

The form variations exhibited in Fig. 2 illustrate that even in the yearling aspen sample, stem form varies. Form variation between species was also observed by MacDonald and Forslund (1986) using this technique. A technique similar to Newnham (1988) and Kozak (1988) in which the exponent of the power function is allowed to vary seems to be an improved approach to estimating individual profiles more closely. By grouping stems with similar characteristics, such as species, age, and site, the average profiles can be plotted to examine how these factors affect the stem form. Once the factors influencing stem form are isolated, they can be incorporated into the exponent of the power function model to produce a more general profile model. The exponent function can perhaps be substituted into the formulae presented earlier to calculate various measures of the stem.

*Volume estimation using the paracone*

The plotting technique was used to examine the optimum position on the stem for obtaining good volume estimates. The position for diameter measurement that provides the best volume estimates using the paracone volume equation (eq. 7) is the position where the average stem profile intersect with the paracone. Figure 1 illustrates a cone, paracone, and paraboloid of equal volume that were plotted using the foregoing technique. The intersection of the three profiles near the 30% position (Fig. 1) implies that a position of form stability exists here, which ensures that similar volume estimates will result using eq. 6 and $D_{0.3}$, regardless of stem form. The 30% position has been recommended by Pollanshütz (1965), Ueno (1978), Schmid-Haas and Winzeler (1981), Forslund (1982), and others as the optimum position to measure an upper diameter for volume estimation. Solving eq. 6 for a given height and diameter at $Y = 0.3$, the volume of the cone is 1.3% less than that of the paracone, and the paracone is 3.6% less than that of the paraboloid. Since stem form often varies from conical to paraboloidal, the paracone volume model (eq. 7) using $D_{0.3}$ and $H$ appears to be a logical average model for the 30% position.

The actual location of intersection of each profile can be calculated by solving a system of two equations using eq. 6 for pairs of profiles with equal volume and form values of $A1$ and $A2$:

$$[19] \quad Y = 1 - \left[\frac{A2(A1 + 2)}{A1(A2 + 2)}\right]^{(A1\,A2/(2A1\,-\,2A2))}$$

The calculated values of $Y$ using eq. 19 are 0.370 for the paraboloid and paracone, 0.333 for the paraboloid and cone, and 0.314 for the paracone and cone. This suggests that perhaps a position slightly higher than 0.300 would provide better volume estimates. The position at 0.333 (or one-third) appears to be a good candidate. It is interesting to note that the British Columbia Forest Service (1948) proposed the formula $V = (3/4)\pi D^2_{0.33}H/4$ to calculate volume. This equation results from eq. 6 using either the cone or the paraboloid at the one-third position. The paracone underestimates the volume of either the cone or paraboloid at this position by 1.9%.

Using Freese's $\chi^2$-test of accuracy (Freese 1960), Table 1 indicates that the best volume estimates are obtained at either the 30 or 33% position regardless of the form of the stem. The best overall form for volume estimations at various measurement positions is the paracone, and the 33% position is a slight improvement over the 30% position. Forslund (1982) and MacDonald and Forslund (1986) showed that the paracone predicts volumes to within 10% using only two

TABLE 3. Averages of height, centre of gravity (COG), centre of mass (COM), form ($A$), and taper ($T$) over each height class of the aspen sample

| Height class | Height (cm) | Actual COG[a] | Calc. COG[b] | Calc. COM[c] | Actual $A_{COG}$[d] | Calc. $A_{COG}$[e] | Calc. $A_{COM}$[f] | Taper[g] |
|---|---|---|---|---|---|---|---|---|
| 1 | 64.0 | 0.334 | 0.321 | 0.274 | 2.01 | 1.79 | 1.72 | 206 |
| 2 | 85.5 | 0.314 | 0.305 | 0.260 | 1.69 | 1.56 | 1.54 | 208 |
| 3 | 101.5 | 0.309 | 0.303 | 0.256 | 1.62 | 1.54 | 1.49 | 224 |
| 4 | 122.6 | 0.304 | 0.299 | 0.250 | 1.55 | 1.49 | 1.42 | 227 |
| All | 93.4 | 0.315 | 0.307 | 0.260 | 1.70 | 1.59 | 1.54 | 216 |

[a]Actual COG was obtained by balancing the stems. The COG for a neiloid, cone, paracone, and paraboloid are 0.200, 0.250, 0.300, and 0.333, respectively, using eq. 10.
[b]Calculated COG from the diameter–height measurements.
[c]Calculated COM from the diameter–height measurements. The COM for a neiloid, cone, paracone, and paraboloid are 0.159, 0.206, 0.257, and 0.293, respectively, using eq. 11.
[d]Calculated form using the actual average COG and eq. 13.
[e]Calculated form using the calculated average COG and eq. 13.
[f]Calculated form using the calculated average COM and eq. 14.
[g]Taper ($T$) was calculated using eq. 15.

stem measurements: $D_{0.3}$ and $H$ in eq. 7. For this sample, similar accuracy was obtained using the paracone and diameter measurements between the 10 and 50% positions. All other forms generally produce poorer estimates in these regions of the stem.

*Quantitative measures of stem form*

The previously defined indicators of stem form will now be used to quantitatively examine the hypothesis of a paraboloidal juvenile profile. Using eq. 12, the sectional form ($A_{SEC}$) given two diameters on the stem was calculated. Diameters separated by 10% of the total height were used in the calculations to determine each sectional form. The averages by height class for each 10% section are presented in Table 2 and progressively vary from more paraboloidal ($A = 2$) in height class 1 to more paraconical ($A = 1.5$) in class 4. Similarly, the visual profiles of Fig. 2, and the COG, COM, and form values shown in Table 3, reflect the same trend over the height classes.

In addition to determining the actual COG by balancing the stems, the COG and COM were also calculated from the diameter–height measurements using calculus and iterative techniques. The actual COM can be obtained by slicing the stem until both halves weigh the same, however this was not done with the aspen sample. The variation in wood density along the length of the stem will cause the actual values of COG and COM to differ from the corresponding calculated values. Notice in Table 3 that the actual COG is consistently higher than the calculated value. The calculated values for COG and COM may be better indicators of stem form than the actual values, since they are based upon the stem measurements, whereas the actual COG value is affected by the wood density.

It is interesting to note that if the form of a stem is a power function, then the same form value ($A$) will be produced by eqs. 13 and 14 using the calculated COG and COM, respectively. As shown in Table 3, there is only a slight variation between the calculated $A_{COG}$ and $A_{COM}$. This variation is due to the deviation of the stem profile from a true power function as can be seen by the basal flare in Fig. 2. Since there are an infinite number of profile functions that can have the same COG but different COM, or visa versa, it is important to consider these two values together when examining the form of the stem profile.

*Quantitative measure of stem taper*

Using eq. 15, a taper value for each stem was calculated. Table 3 summarizes the taper values for each of the four height classes. The increase in the taper values from the smaller to the larger height classes suggests that the taper and form values are inversely related for this sample. As a result, the short and fat stems (low taper value) are more parabolical than the tall and skinny stems (high taper value), which are paraconical. Although the competition and other stresses around each stem were not recorded, the taller stems are likely under more competition and stress than the shorter stems. The progressive increase in the taper values of Table 3, and the progressive increase in basal flare exhibited in Fig. 2, supports this statement. The paraboloidal form of the shorter stems may therefore best represent the initial juvenile stem form before it is influenced by other factors. The truth of the paraboloidal juvenile form hypothesis will be tested further in the future under a controlled greenhouse environment.

## Summary and conclusions

The stem profile examination techniques presented in this paper have illustrated their potential to provide a foundation upon which future studies of the variation in the stem profile can be based. The profile plotting technique allows visual form comparisons to be made between the average profiles of groups of stems. Formulae are also presented to quantify stem form and taper independent of each other to examine form and taper variation between individual and average profiles independent of stem size. Other useful formulae are also presented that allow the calculation of total stem volume, merchantable volume, surface area, and instantaneous slope given the average form of the stem. The instantaneous slope is shown to be directly related to the product of form and taper. The COG and COM of the stem are also presented to introduce some preliminary formulae that will be useful for examining the mechanics of the stem. The collective formulae and techniques presented here may be applicable to the power function models used by Newnham (1988) and Kozak (1988), which allow the exponent to vary along the length of the bole.

Application of the techniques on a sample of 50 yearling aspen supported the hypothesis that juvenile stems begin life as paraboloids and illustrated a progressive form variation

from paraboloidal to paraconical over the variation in height within the sample. If juvenile stems are essentially paraboloidal, then perhaps this fact will provide a valuable alternative method for predicting stem form. Rather than attempting to discover how older stems have arrived at their current form, this approach would examine what factors affect the starting paraboloidal form of juvenile stems and trace the form development as the stems age under a variety of stresses and conditions.

**Acknowledgements**

I wish to thank G.B. MacDonald, D. Morris, D. Weingartner, K. McClain, R. Collier, and the referees for their valuable reviews and input into this paper.

BRITISH COLUMBIA FOREST SERVICE. 1948. Scaling studies. British Columbia Forest Service, Victoria.

CZAPLEWSKI, R.L. 1989. Graphical analysis of stem taper in model building. Can. J. For. Res. **19**: 522–524.

FORSLUND, R.R. 1982. A geometrical tree volume model based on the location of the centre of gravity of the bole. Can. J. For. Res. **12**: 215–221.

FREESE, F. 1960. Testing Accuracy. For. Sci. **6**: 139–145.

GOULD, S.J. 1966. Allometry and size in ontology and phylogeny. Biol. Rev. **41**: 587–640.

GRAY, H.R. 1956. The form and taper of forest-tree stems. Imperial Forest Institute, University of Oxford, Oxford. Inst. Pap. No. 32.

KOZAK, A. 1988. A variable-exponent taper equation. Can. J. For. Res. **18**: 1363–1368.

MACDONALD, G.B., and FORSLUND, R.R. 1986. Application of a geometrical volume equation to species with different bole forms. Can. J. For. Res. **16**: 311–314.

MANDELBROT, B.B. 1982. The fractal geometry of nature. W.H. Freeman and Co., New York.

NEWNHAM, R.M. 1988. A variable-form taper function. Can. For. Serv. Petawawa Natl. For. Inst. Inf. Rep. PI-X-83.

ORMEROD, D.W. 1973. A simple bole model. For. Chron. **49**: 136–138.

POLLANSCHUTZ, J. 1965. Eine neue Methode der Formzahl- und Massenbestimmung stehender Stämme. Mitt. Forstl. Bundesversuchsanst. Wein, **68**: 1–186.

REAL, P.L., MOORE, J.A., and NEWBERRY, J.D. 1989. Principal components analysis of tree stem profiles. Can. J. For. Res. **19**: 1538–1542.

REED, D.D., and BYRNE, J.C. 1985. A simple, variable form volume estimation system. For. Chron. **61**: 87–90.

SCHMID-HAAS, P., and WINZELER, K. 1981. Efficient determination of volume and volume growth. *In* Proceedings of the 17th IUFRO World Congress, 6–12 Sept. 1981, Kyoto, Japan. Japanese IUFRO Congress Committee, Tsuhuba, Ibaraki, Japan. pp. 231–257.

UENO, Y. 1978. On the nature of form factors and its application for stand volume estimation (I). Nippon Rin Gakkaishi, **60**: 87–93.

WINTERS, R.K. 1932. Stem form of white oak. Mich. Acad. **15**: 275–336.

WOOD, G.B., WIANT, H.V., LOY, R.J., and MILES, J.A. 1990. Centroid sampling: a variant of importance sampling for estimating the volume of sample trees of radiata pine. For. Ecol. Manage. **36**: 233–243.

ZEIDE, B. 1987. Analysis of the 3/2 power law of self-thinning. For. Sci. **33**: 517–537.